Test 4, **Math 3107** Name: ____________________________

Spring 2001, Dr. Howard

Instructions: This take-home test is due no later than Wednesday, April 25, at the *beginning* of class. You must *work alone* on all problems. You may use your own text book, your own class notes, your own calculator, and Maple. You may not use any other aids without my explicit permission. Please staple this problem sheet in front of your solutions.

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1. Transform the following system of differential equations into a first-order system of the form $Y' = AY + F(t)$.

$$x'' = 3x - 2y + \sin t$$
$$y'' = 2x + 4y + \cos t$$

2. Complete exercise 28 of section 5.2.

3. In each part below, find a system of linear differential equations $Y' = AY$. For each system, complete the following tasks:
(a). Find the general solution $Y(t)$.
(b). Sketch the phase portrait.

$$\text{(i).}\quad Y' = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} Y \qquad \text{(ii).}\quad Y' = \begin{pmatrix} -1 & 4 \\ -2 & -5 \end{pmatrix} Y$$

4. Consider the system (ii) of problem 3.
   a. Find the particular solution satisfying the initial condition $Y(0) = (2, 0)$.
   b. Use Maple to draw the time series graphs for $0 \le t \le 2$ (for the particular solution obtained in part a). For a sample of the necessary code, see the Maple lecture notes entitled

      ”Graphing solutions of systems of differential equations”.
   c. Use Maple to draw the space curve graph for $0 \le t \le 2$ (for the particular solution obtained in part a).

5. Compute e^{At} for the matrix $A = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$.

6. Use the definition to compute the Laplace transform of the function
$$f(t) = \begin{cases} \cos t & \text{if} \quad 0 \le t \le \pi \\ 0 & \text{if} \quad t > \pi \end{cases}.$$

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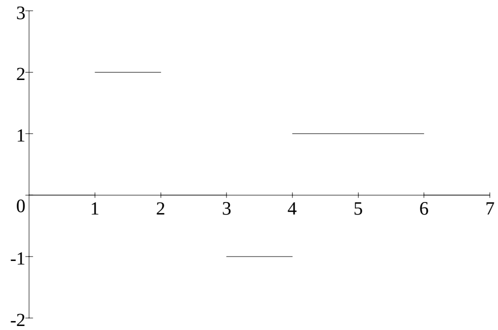
7. Express the function graphed below as a sum of shifted Heaviside functions, i.e. $f(t) = k_1 H(t - a_1) + k_2 H(t - a_2) + \cdots$.



8. In my Maple lecture notes found on the web site, you will find an example demonstrating the solution of a differential equation by Laplace transforms in Maple. Modify that example to solve the following initial value problem:

$$x'' + 4x = 1, \quad x(0) = x'(0) = 0.$$