Test 4, **Math 3107** Name: _____

Spring 2000, Dr. Howard

Instructions: This take-home test is due no later than the beginning of class on Monday, April 24. You must ***work alone*** on these problems. You may use your own text, your own notes, your own calculator, and Maple, but you may use no other aids without my specific permission.

Please show all work and justify all answers. **Staple your work with this she**

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1. Consider the system of differential equations

$$Y' = \begin{pmatrix} 0 & 9 \\ -1 & 6 \end{pmatrix} Y$$

a. Find the general solution $Y(t)$.

b. Sketch the configuration of solutions in the phase plane, indicating any straight-line solutions.

c. $Y(t) = (0,0)$ is an equilibrium solution of this system.

i. Identify it as a node, a saddle point, a center, or a spiral point, and

ii. determine if it is unstable, stable, or asymptotically stable.

2. Consider the system of differential equations

$$Y' = \begin{pmatrix} 7 & 15 \\ -6 & -11 \end{pmatrix} Y$$

a. Find the general solution $Y(t)$.

b. Sketch the configuration of solutions in the phase plane, indicating any straight-line solutions.

c. $Y(t) = (0,0)$ is an equilibrium solution of this system.

i. Identify it as a node, a saddle point, a center, or a spiral point, and

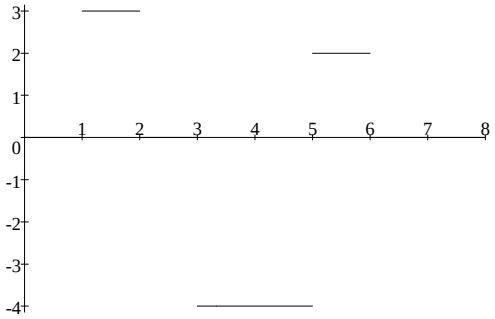
ii. determine if it is unstable, stable, or asymptotically stable.

d. Use Maple to draw the time series solutions, phase plane graph, and space curve corresponding to the initial condition $Y(0) = (1,1)$.

3. Compute e^{At} if A is the matrix $A = \begin{pmatrix} -2 & -5 \\ 4 & 7 \end{pmatrix}$.

4. Compute the Laplace transform of the function $f(t) = \begin{cases} e^{-t} & \text{if } 0 \le t \le 5 \\ 0 & \text{if } t > 5 \end{cases}$.

5. Express the function graphed below as a sum of shifted Heaviside functions, i.e. $f(t) = k_1 H(t - a_1) + k_2 H(t - a_2) + \cdots$.



6. In my Maple lecture notes found on the web site, you will find an example demonstrating the solution of a differential equation by Laplace transforms in Maple. Modify that example to solve the following initial value problem:

$$x'' + 9x = 1, \quad x(0) = x'(0) = 0.$$