

Please show all work and justify all answers on the blank paper provided.

True or False. In problems 1-3, determine if the given statement is true or false. Give reasons to substantiate your answers.

1. If $y_1(x)$ and $y_2(x)$ are solutions of a linear differential equation of the form $L(y) = f(x)$, then $y = ay_1 + by_2$ also is a solution to that differential equation.
2. In the usual mass-spring setup with no damping ($m =$ mass, $k =$ spring constant, $x =$ displacement), if we have an external forcing function of the form $F(t) = F_0 \cos \omega t$ with $\omega = \sqrt{k/m}$, then in the long run the system will tend to toward $x(t) = 0$.
3. The functions $f(x) = 1 + 2x$, $g(x) = 2x + x^2$, and $h(x) = 1 + x$ are linearly independent on the whole real line.
4. Find the general solution of the differential equation $y'' - 2y' - 3y = 0$.
5. Given that $y'' + 6y' + 8y = 2e^{-2x}$ has complementary solution $y_c = c_1e^{-2x} + c_2e^{-4x}$, find the general solution.
6. Find a differential equation for which $y = A \sin 2x + B \cos 2x$ is a solution.
7. A 5-kg mass is hooked on the end of a spring with spring constant 45 N/m (no damping). If the initial displacement is $x(0) = 1$ meter and the initial velocity is $x'(0) = 2$ m/s, find the amplitude of the resulting oscillations (assume no damping/friction).
8. In a mass-spring-dashpot system with $m = 1$ kg and $k = 3$ N/m, what is the minimum value of the damping constant c necessary to prevent the system from oscillating?
9. For the differential equation given below, identify the transient part of the solution and the steady periodic part of the solution.

$$x'' + 2x' + 5x = 5 \cos 3t$$