Test 3, **Math 3107** Name: _____

Spring 2000, Dr. Howard

Please show all work and justify all answers

1. The differential equation $y'' + 5y' + 6y = 3 + 7e^{-2x}$ has complementary solution $y_c = c_1e^{-2x} + c_2e^{-3x}$. Find the ***general solution*** of the differential equation.

2. Given that $y = e^{-x}$ is one solution of the homogeneous differential equation $y''' - 3y'' + 9y' + 13y = 0$, find the general solution.

3. **Mass-spring-dashpot**. A mass of 2 kg is attached to a spring that has spring constant $k = 40\ kg/\sec^2$ and a dashpot having a damping constant of $c = 16\ kg/\sec$. You do not have to solve the corresponding differential equation, but determine if the system is over-damped, under-damped, or critically damped; then in your own words describe the motion of the mass.

4. Transform the following second order system of differential equations into an equivalent first order system of differential equations.

$$x'' = 4x - 3y$$
$$y'' = 2x + 5y$$

5. Write the following system of first order differential equations in matrix form as $\mathbf{x}' = P(t)\mathbf{x} + \mathbf{f}(t)$.

$$x_1' = 3x_1 - 7x_2 + \sin 3t$$
$$x_2' = 5x_1 + x_2 - e^{2t}$$

6. Consider the first order system of differential equations $\mathbf{x}' = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}\mathbf{x}$.

a. Verify that $\mathbf{x}_1(t) = e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{x}_2(t) = e^{2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ are solutions of the system of differential equations.

b. Use the Wronskian to show that $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent.

7. Identify the three ways we have discussed in class for graphically representing solutions of systems of differential equations. Give a brief description of each type of graph.

8. Apply the eigenvalue method to solve the system of differential equations with initial conditions:

$$x_1' = 9x_1 + 5x_2, \qquad x_1(0) = -7,$$
$$x_2' = -6x_1 - 2x_2, \qquad x_2(0) = 9.$$