

Test 3 Solutions,  
Differential Equations,  
Dr. Howard, Spring 1999

1. Linear independence/dependence.

- a. Show **directly** (by looking at linear combinations) that the following functions are dependent on the whole real line:

$$f(x) = 2 - x^2,$$

$$g(x) = 1 + x^2, \text{ and}$$

$$h(x) = 7 + x^2.$$

Be sure to justify your answer.

**Answer:**  $2f(x) + 3g(x) - h(x) = 0$ , so the functions are linearly dependent.

- b. Use the Wronskian to show that the functions are linearly independent:

$$y_1 = e^{2x} \text{ and}$$

$$y_2 = e^{3x}.$$

**Answer:**  $W(y_1, y_2) = \begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} = 3e^{5x} - 2e^{5x} = e^{5x} \neq 0$  so the functions are linearly independent.

- c. Determine if the following functions are linearly dependent or linearly independent:

$$f(x) = e^x,$$

$$g(x) = e^{-x}, \text{ and}$$

$$h(x) = \sinh(x).$$

**Answer:**  $h(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x} = \frac{1}{2}f(x) - \frac{1}{2}g(x)$ , so the functions are linearly dependent.

2. Find the general solution of the differential equation  $2y'' - 7y' - 4y = 0$ .

**Answer:**  $y(x) = c_1e^{-x/2} + c_2e^{4x}$

3. The differential equation  $y'' + 4y = 12x$  has complementary solution  $y_c = c_1 \cos 2x + c_2 \sin 2x$  and a particular solution  $y_p = 3x$ . Use this information to find the solution to the nonhomogeneous differential equation with initial conditions  $y(0) = 5$  and  $y'(0) = 7$ .

**Answer:**  $y = 5 \cos 2x + 2 \sin 2x + 3x$

4. The Method of Undetermined Coefficients (MUC).

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- a. Use the MUC to determine a particular solution of the nonhomogeneous differential equation  $y'' + 4y = 3x$ .

**Answer:**  $y_p = \frac{3}{4}x$

- b. Set up the appropriate form of a particular solution  $y_p$ , using MUC (you do not have to solve for the values of the undetermined coefficients), to the differential equation  $y^{(3)} + y'' = 5e^x + 2x^2$  given that the complementary solution is  $y_c = c_1 + c_2x + c_3e^{-x}$ .

**Answer:**  $y_p = Ae^x + x^2(B + Cx + Dx^2)$

5. Consider a mass-spring-dashpot system for which  $m = 1$ ,  $c = 6$ , and  $k = 13$ .

- a. Find the general solution of the corresponding second order differential equation.

**Answer:**  $x(t) = c_1e^{-3t} \cos 2t + c_2e^{-3t} \sin 2t$

- b. Is the system over-damped, under-damped, or critically damped?

**Answer:** The system is under-damped since  $c^2 - 4km = -16 < 0$  (and solutions oscillate).