

Instructions:

- A. Please work the problems on the blank paper provided.
- B. Staple these sheets in front of your work when finished.
- C. Justify each answer or risk getting no credit.

True or False. In problems 1-5, determine if the given statement is true or false. Give reasons to substantiate your answers.

1. If a population $P(t)$ grows according to the logistic model, then $P(t)$ remains bounded for $t \geq 0$.
2. The model for a free falling object with air resistance does not predict a terminal velocity.
3. By choosing a small step size, Euler's method can be made to exactly measure the solution to any initial value problem.
4. The equation $x^2y'' + 2xy' + 3y = \cos x$ is a linear differential equation.
5. If y_1 and y_2 are solutions of the differential equation $y'' + p(x)y' + q(x)y = 0$, then $w = \alpha y_1 + \beta y_2$ is also a solution.

6. The initial mass of a certain species of fish was estimated at 7 million tons. If undisturbed, the fish would increase at a rate proportional to the total mass, with a proportionality constant of 2/year ($k = 2$). However, commercial fishing at a constant rate removes approximately 15 million tons per year. After approximately how many years will the fish become depleted? (Round your answer to one decimal place).
7. Consider the differential equation $\frac{dx}{dt} = x(x^2 - 2x - 8)$. Sketch the associated phase line and use it to identify each equilibrium as unstable, stable, or asymptotically stable.
8. A motorboat starts from rest with its motor providing a constant acceleration of 3 ft/sec². If water resistance causes a deceleration of $v^2/300$ ft/sec², estimate (to one decimal place) the velocity of the boat 15 seconds after it starts.
9. Use Euler's method with step size $h = 0.1$ to estimate $y(0.2)$ given the initial value problem

$$\frac{dy}{dx} = 2 + x \sin(y), \quad y(0) = 1.$$

(Round your answers to four decimal places).