

Instructions:

- A. Please work the problems on the blank paper provided.
- B. Staple these sheets in front of your work when finished.
- C. Justify each answer or risk getting no credit.

1. (10 points) Determine which of the following functions is/are solution(s) of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$.

- a. $y = 3e^{4x}$
- b. $y = 7 \sin 4x$
- c. $y = 8 \sin 2x$
- d. $y = 5e^{2x}$
- e. All of these are solutions
- f. None of these is a solution

2. (10 points) Describe the difference between a *general solution* for a differential equation and a *particular solution* for the differential equation.

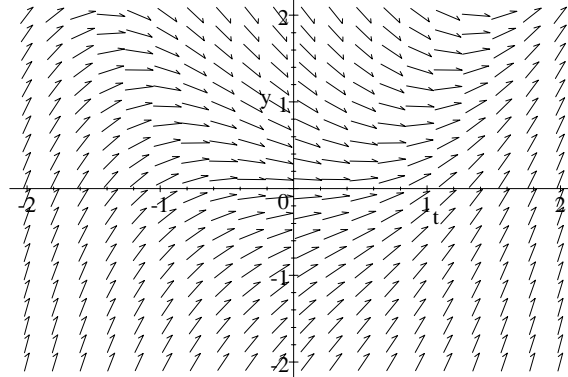
3. (10 points) Several differential equations are given in the table below. Label each differential equation with as many of the following descriptions as appropriate: (i) first order differential equation, (ii) linear differential equation, (iii) ordinary differential equation, (iv) partial differential equation.

$yy' + 3x = 0$	
$x \frac{dy}{dx} + \sin x + y = 0$	
$y'' + 2y' + 3y = \cos x$	
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} = 7$	

4. (30 points) Solve each differential equation. If an initial condition is provided, find the specific solution that satisfies that condition.

- a. $\frac{dy}{dx} = -3xy, y(0) = 2$
- b. $\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = x \cos x$
- c. $xy + y^2 + x^2 - x^2 \frac{dy}{dx} = 0$

5. (10 points) The figure below depicts the direction field for a particular differential equation. Sketch a likely curve through the point (0, 1).



6. (10 points) Assume that your car has a braking system capable of decelerating the vehicle at a rate of 10 meters per second per second (m/s^2). If the brakes are applied when the car has reached a speed of 28 m/s, how far will the car skid before coming to a complete stop?
7. (10 points) Suppose that brine containing 3 kg of salt per liter runs into a tank initially filled with 400 liters of water containing 20 kg of salt. If the brine enters at 10 liters per minute, the mixture is kept uniform by stirring, and the mixture flows out at the rate of 8 liters per minute, find the mass of salt in the tank after 10 minutes. Assume that the tank has the capacity to hold 1000 liters.

Extra Credit:

(10 points) Show that the equation $(2x^2y) \frac{dy}{dx} + 2x^2y + 1 = 0$ is an exact differential equation, then find its general solution.