Test 1, **Math 3107** Name: _______________________

Spring 2000, Dr. Howard

Please work the problems on the blank paper provided. **Staple these sheets in front of your work when finished**.

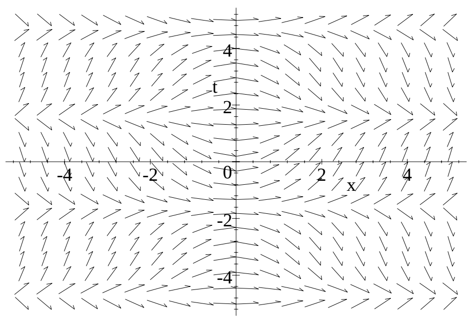
1. Find the general solution of the differential equation $3y + x^4 y' = 2xy$.

2. Find the solution of the initial value problem $y(x+1) + \frac{dy}{dx} = 0,\ y(-1) = 3$.

3. Determine which of the following functions is a solution to the differential equation $y'' + 4y = 0$.

$$y_1 = e^{2x} \qquad y_2 = \sin(2x) \qquad y_3 = \cos(4x)$$

4. The figure below depicts the direction field for a particular differential equation. Sketch a likely curve through the point (0, 4).



5. Suppose that an alligator population P is changing over time at a rate proportional to the square of P. Given that the swamp contained one dozen alligators in 1988, write an differential equation with an initial condition modeling the change of the alligator population with time. ***You do not have to solve the differential equation***.

6. **Short answer**. Cite one of the reasons given in class for which the logistic population model $dP/dt = aP - bP^2$ may be favored to the model for constant relative growth rate: $dP/dt = kP$.

7. Consider the differential equation $\frac{dx}{dt} = 3x - x^2$.

a. Sketch the associated phase line.

b. For the initial condition $x(0) = 5$, determine whether $x(t)$ is increasing, decreasing, or neither.

c. For the initial condition $x(0) = 5$, find $\lim_{t\to\infty} x(t)$.

8. A 120-gallon tank initially contains 90 pounds of salt dissolved in 90 gallons of water. Brine containing 2 pounds per gallon of salt flows into the tank at the rate of 4 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 3 gallons per minute. How much salt is in the tank after 10 minutes?