Test 1, Differential Equations
Dr. Howard, Spring 1999

Test 1 solution key, Spring 1999 Differential Equations

1. Indicate which of the given functions is (are) solution(s) to the differential equation $y'' - 4y' - 6y = 0$.

$$y_1 = e^{-2x} \quad y_2 = e^{-3x} \quad y_3 = e^{3x} \quad y_4 = e^{2x}$$

Answer: None are solutions.

2. Find the solution of the initial value problem $\frac{dy}{dx} = \frac{5}{x^2+1}$, $y(0) = 0$.

$\frac{dy}{dx} = \frac{5}{x^2+1}$, Exact solution is : $y(x) = 5\arctan x + C_1$

$y(0) = 0 \Rightarrow 0 = 5\arctan 0 + C_1 \Rightarrow C_1 = 0 \Rightarrow \boxed{y(x) = 5\arctan x}$

3. Find the general solution of the first order linear differential equation $y' + 2y = 5e^{-2x}$.

Answer: $y(x) = 5e^{-2x}x + e^{-2x}C_1$

4. Suppose that a ball is thrown straight upward from the ground with an initial velocity of 96 ft/sec. What is the maximum height attained by the ball (assuming no air resistance)? After how many seconds does the ball hit the ground?

$y(t)$ = height in feet t seconds after the ball is thrown

$y(t) = -16t^2 + 96t$

Maximum height: Velocity = 0 $\Rightarrow t = 3$, and $\boxed{y(3) = 144 \text{ feet}}$.

Time when it hits the ground: $y(t) = 0$. Solution is : $t = 0$, or $t = 6$. Thus, the **ball hits the ground after 6 seconds**.

5. Given that the differential equation $\frac{dy}{dx} = x^2 - y$ has the slope field below, sketch solution curves through the indicated points.

Sorry, this solution is not available.

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6. A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F. After 30 minutes, the temperature of the cake is 140°F. When will the temperature of the cake be 100°F?

Answer: Approximately 66.67 minutes.

7. Write down a differential equation modeling a spruce budworm population that incorporates the following assumptions:

- for suitably small population sizes, the number of budworms will increase;
- if the population exceeds the natural capacity of the local environment, the number of budworms will decrease; and
- the local environment provides enough resources to sustain the budworm population at a steady level of 20 units of budworms.

Note: your differential equation will have one parameter remaining undetermined (the *r* we had in class).

Answer: $\frac{dP}{dt} = rP\left(1 - \frac{P}{20}\right)$.

8. Find by inspection two different solutions of the initial value problem

$$\frac{dy}{dx} = 3y^{2/3}, \; y(0) = 0.$$

Why does the existence of these *two* solutions not contradict the Existence/Uniqueness theorem?

Please see the text book for the answer.