Taylor's Theorem and Applications

1. Find the second degree Taylor polynomial $P_2(\mathbf{x}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \bullet (\mathbf{x} - \mathbf{a}) + (\mathbf{x} - \mathbf{a})^T H(\mathbf{a})(\mathbf{x} - \mathbf{a})$ for the following:
   a. $f(x,y) = x^3 + 3xy + y^3$; $\mathbf{a} = (-1,-1)$
   b. $g(x,y,z) = x^2yz + \cos(xy)$; $\mathbf{a} = (1, \pi/3, 3)$

2. Find all critical points for each given function and for each critical point **a**,
   (i). evaluate the Hessian matrix $H(\mathbf{a})$,
   (ii). find the eigenvalues of $H(\mathbf{a})$, and
   (iii). determine if the $x^T H(\mathbf{a})x$ is positive definite, negative definite, or indefinite, and
   (iv). classify **a** as a relative minimum, relative maximum, or saddle point.
   a. $f(x,y) = x^3 + 3xy + y^3$
   b. $g(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$