Test 2, Math 3105 Name: _____
Fall 2001, Dr. Howard

Please show all work and justify all answers on the blank pages provided.

---

1. The path of a moving particle is parameterized as $x = e^t \cos t$, $y = e^t \sin t$, and $z = e^t$.

a. Find the speed of the particle as a function of time.

b. Find the arc length of the path traversed on the interval $[0, t]$.

c. Find the tangental and normal components of acceleration as functions of time.

2. Find **v** and **a** in terms of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ if a particle moves in such a way that

$$r = 3(1 - \sin\theta), \qquad \frac{d\theta}{dt} = 3.$$

3. If $f(x,y,z)$ is a differentiable scalar field, explain what the quantity $\nabla f \bullet (-\mathbf{k})$ represents.

4. Compute *grad f* for $f(x,y,z) = \tan xy + 3z$.

5. Find a vector that is normal to the ellipsoid $x^2 + 2y^2 + z^2 = 9$ at the point $(1, -1, \sqrt{6})$.

6. Let $\mathbf{F} = x\mathbf{i} + y^3\mathbf{j} + 2\mathbf{k}$.

a. Find the general equation of a flow line.

b. Find the flow line through the point $(1, 1, 0)$.

7. Find *div* **F** given that $\mathbf{F} = \sec(xyz)\mathbf{i} + e^x\mathbf{j} + \mathbf{k}$.

8. Give an example of a field with a constant negative divergence.

9. Find *curl* **F** given that $\mathbf{F} = x^2y\mathbf{i} + 2z\mathbf{j} + xy^2z\mathbf{k}$.

10. Check to see whether or not each of the following functions satisfies Laplace's equation, $\Delta f = 0$.

$$f(x,y,z) = x^2 + y^2 - 2z^2$$
$$g(x,y,z) = x^2 - y^2 + z^2$$