Test 2, **Math 3105** Name: ________________________

Fall 2000, Dr. Howard

Please show all work and justify all answers on the blank sheets provided.

---

1. Determine a unit vector tangent to the path $c(t) = (t, t^2, t^3)$ at the point $(2, 4, 8)$.

2. The position in space of a certain particle is given by $c(t) = t\,\mathbf{i} + \sin(2\pi t)\,\mathbf{j} + \cos(2\pi t)\,\mathbf{k}$.

a. Find the velocity vector of the particle at $t = 1$.

b. Find the acceleration vector of the particle at $t = 1$.

c. Find the arc length traversed by the particle during the time interval $0 \le t \le 1$.

3. Verify that $c(t) = t^2\,\mathbf{i} + (2t - 1)\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$, $t > 0$, is a flow line of the velocity vector field $\mathbf{F}(x, y, z) = (y + 1)\,\mathbf{i} + 2\,\mathbf{j} + \frac{1}{2z}\,\mathbf{k}$.

4. Let $c(t)$ be a flow line of a gradient field $\mathbf{F}(x, y, z) = -\nabla V$ for some scalar field $V(x, y, z)$. Prove that $V(c(t))$ is a decreasing function of t.

5. Let f and g be C^1 scalar fields with domain R^3. Find an equivalent expression for $\nabla(fg)$ and prove that it is correct (the sought expression should come from the identities studied in chapter 4 of the text).

6. Compute the divergence of the vector field $\mathbf{F}(x, y, z) = \sin(xy)\,\mathbf{i} + (z - \cos(x^2 y))\,\mathbf{j} + (z^3 + xz)\,\mathbf{k}$.

7. Compute the curl of the vector field $\mathbf{F}(x, y, z) = xyz\,\mathbf{i} + xy^2z^2\,\mathbf{j} + x^2y\,\mathbf{k}$.

8. Show that $f(x, y, z) = 2xy - 3z$ satisfies Laplace's equation.

9. Let f be a scalar field with $f(x, y, z) = \frac{x^2 z}{y}$ and let $c(t)$ be the path $c(t) = (t, 2t^2, 3t)$ for $0 \le t \le 1$. Compute $\int_c f\,ds$, the path integral of f along c.