

## Chaos Assignment

July 5, 2000

1. Complete the analysis of the logistic map  $f(x) = rx(1 - x)$  by responding to the following:
  - a. Use calculus techniques to find the vertex of the graph of  $f$  in terms of  $r$ .
  - b. Verify by hand calculation that  ${}^2f(x) = -r^3x^4 + 2r^3x^3 - r^2(r + 1)x^2 + r^2x$ .
  - c. Using long division, complete the division:  
 $[r^3x^3 - 2r^3x^2 + r^2(r + 1)x - (r - 1)(r + 1)] \div [rx - (r - 1)]$ .
  - d. If  $x_0$  lies on a nontrivial two-cycle of  $f$ , use the Chain Rule Along a Cycle to verify that  
 $({}^2f)'(x_0) = -r^2 + 2r + 4$ .
  
2. Consider the quadratic map  $Q_c(x) = x^2 + c$  with domain  $(-\infty, \infty)$  in responding to the following:
  - a. Determine the conditions on  $c$  necessary for  $Q_c$  to have at least one fixed point.
  - b. Using the parameter values for  $c$  determined in part a, analyze the attraction/repulsion characteristics of all fixed points. Your answer should be broken into cases that depend on the value of  $c$ .
  - c. For what values of  $c$  does  $Q_c$  have a nontrivial two-cycle?
  - d. For the parameter values determined in part c, analyze the two-cycle to determine if it is attracting or repelling. Your answer should be broken into cases that depend on the value of  $c$ .