

1. Let $f_r(x) = rz(1 - z)$. Prove that if $r \neq 0$ (but possibly complex) and $|z| > \frac{1}{|r|} + 1$, then $|f_r(z)| > |z|$. Use this to give an escape criterion for the logistic family. That is fill in the blank in the following theorem, then supply the proof of the theorem.

Theorem:

If $f_r(z) = rz(1 - z)$, $r \neq 0$, $|z| > \frac{1}{|r|} + 1$, and _____ then $|f_r^n(z)| \rightarrow \infty$ as $n \rightarrow \infty$.