

**Please show all work and justify all answers.**

1. Recall the "doubling function"

$$D(x) = 2x \bmod 1 = \begin{cases} 2x & \text{if } 0 \leq x < 1/2 \\ 2x - 1 & \text{if } 1/2 \leq x < 1 \end{cases}$$

Discuss the orbit behavior for the seeds  $x_0 = 1/7$  and  $x_0 = 1/8$ .

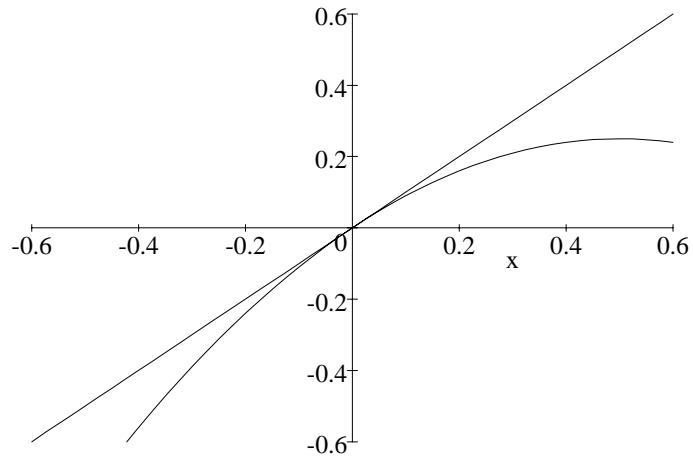
2. Consider the function  $f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2$ .

a. Find all fixed points of  $f$ .

b. Classify each fixed points as attracting, repelling, or neutral.

3. Consider the function  $f(x) = x - x^2$ .

a. Sketch the web diagram associated with the seed  $x_0 = 1/2$  using the graph below.



b. Give a complete orbit analysis for  $f$ . That is, describe what happens to all orbits in terms of their seed values. (Suggestion: it may help you to draw a more complete graph of  $f$ .)

4. Consider the curve obtained by the iteration started below.

Initiator:



Generator:



- a. Sketch the next stage of the iteration.
- b. Determine the similarity dimension (fractal dimension) of the limiting fractal curve.

5. Recall the "tent map"  $T(x) = \begin{cases} 2x & \text{if } 0 \leq x < 1/2 \\ 2 - 2x & \text{if } 1/2 \leq x \leq 1 \end{cases}$ .

a. Give a piecewise definition for  ${}^2T(x)$ .

b. Find a 2-cycle of  $T$ . (Hint: it may help to draw the graph).