

Chaos Assignment

June 21, 2000

Consider the "doubling map"

$$D(x) = \begin{cases} 2x & \text{if } 0 \leq x < 1/2 \\ 2x - 1 & \text{if } 1/2 \leq x < 1 \end{cases}$$

with domain $[0, 1)$. Equivalently, this function can be defined as $D(x) = 2x \bmod 1$. Use this function in completing the following.

1. By hand, calculate the first 15 terms in the orbit of $1/9$. Express each term as a fraction in reduced form. What is the long term forecast for this orbit?

Answer: The orbit is a 6-cycle,

$$\frac{1}{9} \rightarrow \frac{2}{9} \rightarrow \frac{4}{9} \rightarrow \frac{8}{9} \rightarrow \frac{7}{9} \rightarrow \frac{5}{9} \rightarrow \frac{1}{9} \rightarrow \frac{2}{9} \rightarrow \frac{4}{9} \rightarrow \frac{8}{9} \rightarrow \frac{7}{9} \rightarrow \frac{5}{9} \rightarrow \frac{1}{9} \rightarrow \frac{2}{9} \rightarrow \frac{4}{9} \rightarrow \dots$$

2. Using your calculator, compute the first 25 terms in the orbit of $1/9$. Include ten decimal places for each term. What appears to be the long term forecast for this orbit?

Answer:

0.1111111111	0.1111111111	0.1111111111	0.1111111115	0.1111111344
0.2222222222	0.2222222222	0.2222222222	0.2222222230	
0.4444444444	0.4444444444	0.4444444445	0.4444444459	
0.8888888889	0.8888888889	0.8888888889	0.8888888918	
0.7777777778	0.7777777778	0.7777777779	0.7777777836	
0.5555555556	0.5555555556	0.5555555557	0.5555555672	

3. Write a paragraph comparing/contrasting your observations from parts 1 and 2.

Answer: When the calculations are performed using exact arithmetic (the fractions), we observe that the orbit is a 6-cycle. However, when the calculations are performed on the calculator, some roundoff error occurs. As a result, we do not get a perfect 6-cycle.