Chaos Assignment
June 27, 2000

For each of the following functions, find all fixed points and classify them as attracting, repelling, or neutral. Use the Attraction Theorem and the Repelling Theorem when they apply; in other cases, use a web diagram to assist you in drawing your conclusion.

1. $f(x) = x^2 - x/2$

Answer: To determine the fixed points, we solve the equation

$$x^2 - \frac{x}{2} = x$$

$$x^2 - \frac{3}{2}x = 0$$

$$x(x - \frac{3}{2}) = 0$$

$$x = 0 \text{ or } x = \frac{3}{2}$$

To determine whether the fixed points are attracting, repelling, or neutral, we check the derivative $f'(x) = 2x - 1/2$.

$$|f'(0)| = |-1/2| = 1/2 < 1, \text{ so 0 is an attracting fixed point}$$

$$|f'(3/2)| = |5/2| = 5/2 > 1, \text{ so 3/2 is a repelling fixed point.}$$

2. $f(x) = x(1 - x)$

Answer: 0 is the only fixed point. $f'(0) = 1$, so the Attraction Theorem is inconclusive. Web diagrams indicate the 0 is attracting from the right and repelling from the left, so it is a *neutral* fixed point.

3. $f(x) = \frac{\pi}{2}\sin x$

Answer: 0 is the only fixed point. $f'(0) = \frac{\pi}{2} > 1$, so we have a repelling fixed point.

4. $T(x) = \begin{cases} 2x & \text{if } x \le 1/2 \\ 2 - 2x & \text{if } x > 1/2 \end{cases}$

Answer: 0 and 2/3 are fixed points. $T'(0) = 2 \Rightarrow$ 0 is a repelling fixed point.
$T'(2/3) = -2 \Rightarrow$ 2/3 is also a repelling fixed point.

5. $f(x) = -x + x^3$

Answer: The fixed points are 0 and $\pm\sqrt{2}$. $f'(0) = -1$, so the Attraction Criterion doesn't apply; the orbit diagram obtained with WINFEED suggests that 0 is weakly attracting.
$f'(\pm\sqrt{2}) = 5 \Rightarrow$both $\sqrt{2}$ and $-\sqrt{2}$ are repelling fixed points.