

MAT 131 Test 3

Fall 1997

Show all work for credit. Do all your work neatly on the paper provided. Write your name on each sheet you turn in. I will **not** grade any work done on the test sheet. When you are finished turn in all sheets including the test. Good Luck.

1. A sailor standing on the edge of a dock 15 feet above the lakes surface is pulling in her boat by means of a line attached to the boats bow. She pulls in the line at a rate of  $5\frac{ft}{sec}$ . How fast is the boat approaching the foot of the dock when the boat is 20 feet away.
2. Given  $f(x) = x^3 - 12x + 10$ 
  - a. Find the intervals on which  $f$  is increasing.
  - b. Find the intervals on which  $f$  is decreasing.
  - c. Find the intervals on which  $f$  is concave up.
  - d. Find the intervals on which  $f$  is concave down.
3. Find all critical points of  $f(x) = \frac{x^2 - 1}{x^2 - x - 2}$ .
4. How many real roots does  $\frac{x^3}{3} - \frac{x^2}{2} - 2x + 1 = 0$  have?
5. Find all horizontal and vertical asymptotes of  $y = \frac{x^2 - 4}{x^2 - 3x + 2}$ .
6. Graph  $f(x) = \frac{x^2 + 1}{x^2 - 1}$ .
7. Given  $g(x) = \frac{1}{x^2 + 1}$ ,  $g'(x) = \frac{-2x}{(x^2 + 1)^2}$ , and  $g''(x) = \frac{2(3x^2 - 1)}{(x^2 + 1)^3}$  graph  $g$ . Please label all max's/min's and points of inflection.
8. A cylindrical tin can is to hold  $50cm^3$  of fluid. How should it be constructed in order to minimize the amount of material needed in its construction? (Note: Tin cans have tops and bottoms)
9. Given an example of a function and an interval for which the mean value theorem does not hold and explain why not.